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QUADRUPOLE MOMENTS OF HEAVY NUCLEI

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Abstract of article may be omitted

Summary: The drop model (Gamov's model) of a nucleus, explains ~~considers~~ successfully all aspects of nuclear energy, including non-central forces of interaction, and permits one to obtain, relative to the order of magnitude, the correct values of positive quadrupole moments for a large majority of nuclei. A small number however of nuclei with negative quadrupole moments can not be brought into the general scheme with equal certainty.

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In a preceding work [1] we indicated that positive quadrupole moments of nuclei can be explained, within the framework of the so-called drop model (Gamov's model), by non-central nuclear forces, ~~It was shown~~ and showed that non-central forces can be added to the drop model by introducing an additional "surface tension" which depends upon the angle between the spin axis and the norm to the surface. We considered the stable shape of the nucleus to be an ellipsoid of revolution ~~rotation~~ of such eccentricity as determined by: the condition of minimum total energy: ordinary surface energy, the surface energy dependent upon ~~conditioned by~~ non-central forces, and Coulomb energy.

The present work evaluates the ratio α_2/α_0; the alpha's α_2 and α_0 are "phenomenological" constants in the energy expression per unit area of the surface $\alpha_0 + \alpha_2 \cos^2(n, z)$, which is based on the condition that the central and non-central forces are of the same order of magnitude. The calculated theoretical values, with the aid of this ratio, of the quadrupole moments of heavy nuclei are of the same order of magnitude as observed experimental values.

The question concerning the sign of the quadrupole moment is also discussed. In selecting the sign of non-central forces, the exchangeable part of the general potential energy of the nucleus, in accordance with the deuteron theory and with non-central nuclear forces, is noted to be negative and with the cause of the tendency of the nucleus to ~~form a cigar shaped oblate~~ become elongated. Nevertheless, if there is an unequal number of protons and neutrons in the nucleus, then there is also a "direct" non-central force of interaction, which is a repulsion and favors a negative quadrupole moment. This "direct" interaction is proportional to the square of the spin number (and the square of ~~an~~ the isotopic number), while the "exchange[redacted]" reaction is proportional to the spin number.

The magnitude of the negative quadrupole effect, produced through "direct" non-central interaction is ~~governed by~~ dependent upon the "exchange" reaction.

Thus, the non-central forces cause positive quadrupole moments, as disclosed experimentally for most nuclei. Occasionally the observed negative quadrupole moments are smaller in magnitude than the positive ones and can be explained by other reasons. A small negative quadrupole effect is possibly caused by the rotation of the nucleus.

I. Order of magnitude of the quadrupole moment

The nucleus' model is taken to be a uniformly charged ellipsoid of ~~rotation~~ revolution whose size equals that of an orbit of radius $r_0A^{1/3}$. An a-priori possibility is that the nucleus is ~~cigar shaped~~ elongated, besides flat and oblate, along the spin axis. Further, the spin density of particles (the difference of the density of particles with spins, parallel and anti-parallel to the z-axis, is equal to the resultant spin) is assumed to be stable in the entire volume of the nucleus. One can regard the spins of all particles as parallel to the z-axis; but in addition to this, the mean value of the spin particle is equal to i/A (i is the spin of the nucleus in units of $\hbar/2$).

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note: this is the ordinary letter z (zee).

The non-central nuclear forces, proportional to $(\vec{\sigma}_1 r)(\vec{\sigma}_2 r)/r^2$, form an additional surface energy. In our classical model the energy ~~of~~ per unit surface ~~governed by~~ due to these non-central forces can be written as:

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$$\alpha_2 \cdot \cos^2(n, z) \qquad (1)$$

where n is the normal to the surface; α_2 is ~~a~~ some phenomenological quantity such as the ordinary surface tension α_0 of a drop nucleus. The eccentricity ε of the nucleus is obtained from the condition that the total energy be a minimum and is determined by formulas derived from a previous work [1], for $|\varepsilon^2| \ll 1$ from the following equation:

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$$\frac{\alpha_2}{\alpha_0}\left(\frac{32}{45} + 0.237\,\varepsilon^2 + \cdots\right) + B\frac{Z^2}{A}\left(\frac{4}{45}\varepsilon^2 + \cdots\right) - \frac{16}{45}\varepsilon^2 - \cdots = 0 \qquad (2)$$

where $B = (3e^2/5r_0)/4\pi r_0^2\alpha_0 \qquad (3)$

~~Adapting~~ Setting $r_0 = 1.4 \cdot 10^{-13}$ cm and $4\pi r_0^2 = 14$ Mev [2], we obtain:

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$$\varepsilon^2 \approx (\alpha_2/\alpha_0)\left[0.5 - 0.01Z^2/A\right]^{-1} \qquad (4)$$

The quantity α_2 can be evaluated from the fact that the central and non-central forces are of the same order of magnitude, in agreement with existing theories of nuclear forces.

If only central forces are present, the ratio of surface energy to potential energy due to these forces is of the same order of magnitude as the ratio of the effective radius r_0 of activity of nuclear forces to the nuclear radius R. Assuming the potential energy to be proportional to the number of particles, we obtain:

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$$4\pi r_0^2 A^{2/3}\alpha_0 / E_0 A \approx r_0/R \qquad (5)$$

where $R = r_o A^{1/3}$ and E_o is the average potential energy per particle.

Hence:

$$4\pi r_o^2 \alpha_o \approx E_o \qquad (6)$$

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The surface energy due to non-central forces has, according to (5) [1] for the case of a spherical nucleus, the value

$$U_2 = (4\pi/3) \cdot r_o^2 A^{2/3} \alpha_2 \qquad (7)$$

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Because of the small eccentricities of nuclei, the value of U_2 is of the same order of magnitude as most non-spherical nuclei. There is a relationship between the surface energy and the total potential energy due to non-central forces, which is similar to (5):

$$(4\pi/3) r_o^2 A^{2/3} \alpha_2 / E_2 i \approx r_o / R \qquad (5a)$$

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where E_2 is the average potential energy per particle for non-central forces, under the condition that the spins be parallel ($i = A$). The factor i is in expression (5a) instead of A in (5), because each particle has only the spin i/A.

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Central and non-central forces are of the same order of magnitude $E_o \approx 3E_2$ and on the basis of (5) and (5a) we have

$$\alpha_2 \; \alpha_o \approx i/A \qquad (8)$$

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Hence instead of (4) we get:

$$\varepsilon^2 \approx (i/A)(0.5 - 0.01 Z^2/A)^{-1} \qquad (9)$$

With this value of eccentricity we estimate the nucleus' quadrupole moment for a uniformly-charged ellipsoid of ~~rotation~~ revolution with charge Z, by means of the formula following:

$$Q = (2/5) r_o^2 Z A^{2/3} \varepsilon^2 (1 - \varepsilon^2)^{-2/3} \qquad (10)$$

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We obtain

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$$Q \approx r_0^2 Z (i/A^{1/3}) \cdot [1 - 0.02 Z^2/A]^{-1} \qquad (11)$$

where $r_0^2 \sim 10^{-26}$ cm^2. For existing heavy nuclei this formula gives the quadrupole moments of the order 10^{-24}cm^2, which is in proper agreement with experimental facts (see chart 1).

CHART I

Nucleus Quadrupole Moments

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(+) signs

Nucleus	Q 20^{24} cm	Nucleus	Q 10^{24} cm
$_{29}Cu^{63}$	$-0,1$	$_{63}Eu^{153}$	$+2,5$
$_{30}Cu^{65}$	$-0,1$	$_{70}Yb^{173}$	$+3,9$
$_{31}Ga^{69}$	$+0,2$	$_{71}Lu^{175}$	$+5,9$
$_{31}Ga^{71}$	$+0,13$	$_{71}Lu^{176}$	$+7$
$_{33}As^{75}$	$+0,3$	$_{73}Ta^{181}$	$+6$
$_{36}Kr^{83}$	$+0,15$	$_{75}Re^{185}$	$+2,8$
$_{49}In^{115}$	$+0,84$	$_{75}Re^{187}$	$+2,6$
$_{53}J^{127}$	$+0,8$	$_{80}Hg^{201}$	$+0,5$
$_{63}Eu^{151}$	$+1,2$	$_{83}Bi^{208}$	$-0,4$

In view of the classical and approximate nature of the examined problem, formula (11) does not pretend to be in a close agreement with experimental results, but only gives the correct order of magnitude. As a result of this classical examination, the non-disappearance of the quadrupole moment in (11) for the spins equal to 1/2 ($i = 1$) also follows.

2. The Sign of the Quadrupole Moment of Nuclei.

According to formulas (4) and (10), the sign of quadrupole moments is determined by the sign of $\varkappa_2$ (the denominator of these expressions is positive). Positive quadrupole moments (stretched out along the spin axis of the ellipsoid of ~~rotation~~ revolution) correspond ~~with~~ to positive values of $\varkappa_2$; that is, to positive surface energy (negative potential energy), due to non-central forces, which holds true if the sign of the non-central interaction is chosen in accordance with the deuteron theory [3, 4].

The surface energies which we introduced in testing the drop model are dependent, just as in the case of a macroscopic fluid drop, upon forces having the character of saturation, which causes the energy of the system to be proportional to the number of interacting particles. We took advantage of this fact in (5), (5a) and (8), in the determination of the magnitudes, α_0, α_2.

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The exchange forces discussed in theories of nuclear forces possess saturation, for example, as in the symmetrical meson theory, [8] in which the function of interaction of two nuclear particles contains an operator $(\vec{\tau}_1 \vec{\tau}_2)$ operating on the "charged" coordinates m_τ, m_τ ~~of the~~ particles. We shall show that such an operator, under certain conditions leads to certain mathematical terms in the expression for contact energy that favor negative quadrupole moments, besides terms that give rise to saturation and create a positive quadrupole effect.

We shall illustrate the non-central interaction of two nuclear particles as follows:

$$U(1,2) = -(\vec{\tau}_1 \vec{\tau}_2)\, V(1,2) \qquad (12)$$

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where

$$V(1,2) = f(r)\,(\vec{\sigma}_1 r)(\vec{\sigma}_2 r)/r^2 \qquad (13)$$

In agreement with the deuteron theory, the function f(r) should be negative. Moreover, U (1,2) is negative for conditions antisymmetrical in "charged" coordinates (for instance, the basic triplet s-d deuteron condition) and is positive for conditions symmetrical in "charged" coordinates (2 protons or 2 neutrons). If the nuclear model has particles of spin parallel to the z-axis, then expression

ordinary typed letter "zee".

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NOTE! "ordinary zee"

(13) assumes the form:

$$V(1,2) = f(r)\cdot\cos^2\vartheta \qquad (14)$$

where (theta) ϑ is the angle formed by the radius vector of two particles with the z-axis. The wave function of the nucleus can be introduced in the form of a determinant, composed in the usual way from the functions $\psi_1(x_1), \psi_2(x_2), \ldots \psi_N(x_N)$, where x_i is understood to be a combination of spatial and "charged" coordinates (N is the total number of particles).

The potential energy of a nucleus according to the interaction (12) is:

$$U = \tfrac{1}{2}\sum_i\sum_k \iint \psi_i^*(1)\psi_k^*(2)\,U(1,2)\,\psi_i(1)\psi_k(2)\,dx_1 dx_2 -$$

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$$-\tfrac{1}{2}\sum_i\sum_k \iint \psi_k^*(1)\psi_i^*(2)\,U(1,2)\,\psi_i(1)\psi_k(2)\,dx_1 dx_2 = J+K \qquad (15)$$

Operator $\vec{\tau}$ can be introduced by means of Pauli matrices, effecting the wave functions p, n of "charged" coordinates in the same way as the spin operator $\vec{\sigma}$ influences the spin functions α, β. By ~~Examining~~ examining the individual wave functions ψ_i of particles in the form of products of spatial wave functions and functions p or n, corresponding to the "proton" or "neutron" condition of the particles, and by denoting the spatial wave functions of protons and neutrons accordingly by ψ_p ψ_n, we obtain, in accordance with (12) and (15), the following expressions for "exchange" interaction K and "direct" interaction J:

$$K = \qquad (16)$$

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$$J = \qquad (17)$$

Included here are ~~Hence~~ the "exchange" integral K and the "combined" densities of protons and neutrons:

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(18)

The "direct" integral J has the usual densities:

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(19)

The "exchange" integral K is negative and is dependent upon the previously considered positive quadrupole moment. The "direct" interaction J is positive for a non-disappearing difference of proton and neutron density; the corresponding surface energy is negative and forms a negative quadrupole effect. One can evaluate this effect, if one assumes that the interaction of type J, occurring ~~leading~~ also for central forces and leading to "non-saturation", does not actually disturb the dependency of the radius of the nucleus upon the atomic weight: $R = r_0 A^{1/3}$.

If we assume (as before) ~~, as before,~~ the non-central and central forces to be of the same order of magnitude and substitute the densities ρ_p and ρ_n in (17) corresponding to the spin densities, which gives the additional factor i^2/A^2, and if we let the function f(r) be the Yukawa potential:

kappa

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$$f(r) = -g^2 \cdot e^{-\kappa r}/r \qquad (20)$$

and disregard the surface effect, then we shall obtain for integral J with the help of (20) and (14) the following values:

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$$J = (i^2/2A^2) \cdot (A-2Z)^2 E_0/A \; ; \; E = g^2/r_0 \; ; \; r_0 \approx 1/\kappa \qquad (21)$$

kappa

We note, by the way, that this integral for the case of central forces is equal to $(3/2)E_0 \cdot (A - 2Z)^2/A$ and corresponds with Bethe's

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semi-empirical formula [6].

The surface energy corresponding ~~with~~ to (21) (negative) is derived by multiplying (21) by $r_0/R = A^{-1/3}$, The ratio α_2/α_0 dependent, on the basis of (4) and (10), upon the quadrupole moment, will diminish in comparison with (8) because of the inequality in the numbers of protons and neutrons in the nucleus to a ~~magnitude~~ value of the order $(i^2/A^2)\,(A-2Z)^2 \cdot A^{-2}$. The ratio of the magnitude of negative quadrupole moment to the magnitude of positive effect produced by non-central forces is of the order $(i/A)\,(A-2Z)^2\,A^{-2} \ll 1$. An exact estimate of integral (17) for an ellipsoid of ~~rotation~~ revolution, in view of non-saturation, would probably give a larger value of the negative effect, though the negative effect is considerably smaller than the positive.

In this way the non-central nuclear forces always give a positive quadrupole moment. ~~The indicated~~ Chart 1 (see footnote 1) illustrates that in most nuclei the experimentally measured quadrupole moment is actually positive and coincides as to order of magnitude with the values calculated by formula (11). Now and then negative quadrupole moments are encountered which are smaller as to order of magnitude than the positive moments (10^{-25} cm^2 instead of 10^{-24} cm^2). The smallness of negative quadrupole moments follows also from the results of new experimental works [9]. Apparently, this ~~effect~~ is due to the action of other causes and occurs upon the disappearance of an effect caused by non-central nuclear forces.

If we remain in the framework of the drop model, it is necessarry

Footnote 1: The chart is taken from the work of Inglis [7], where one can find references to corresponding sources. The value of the quadrupole moment for Ta^{181} was taken from the work of Schmidt [8].
~~quadrupole moment for Ta~~

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to inspect the influence of nuclear rotation upon the quadrupole moment. Because of rotation, the nucleus along the spin axis becomes quite flattened. Employing the $\boxed{\text{term}}$ kinetic energy of the rotator $(h^2/2I)\,K(K+1)$ (K is the spin quantum number and I is the moment of inertia) in our model of the nucleus, namely, an ellipsoid of ~~rotation~~ revolution, we obtain:

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$$E_{kinetic} = (5/4)(\hbar^2 K(K+1)/Mr_o^2)\, A^{-\frac{5}{3}}\, \kappa^{-\frac{2}{3}} \qquad (22)$$

kappa

kappa

where κ = b/a is the ratio of the axes of the ellipsoid (a is the axis of rotation and M is the mass of the nuclear particle). If we examine this kinetic energy for minimum total energy as a function of the parameter κ, we shall obtain for the condition of smallness of eccentricity ($|\varepsilon^2| \ll 1$) an additional term in the left part of equation (2):

$$-2.4\,K(K+1)\cdot A^{-7/3}\cdot\left[1+\tfrac{5}{6}\varepsilon^2+\cdots\right]$$

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In the absence of the influence of non-central forces, we obtain a negative quadrupole moment:

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$$Q' = -2.7\, r_o^2\, Z A^{-5/3}\, K(K+1)\left[1-0.02\,Z^2/A\right]^{-1} \qquad (23)$$

As expression (23) shows, the effect of rotation is not sufficiently large to explain the negative quadrupole moments for any suitable choice of K. Thus, in obtaining the quadrupole moment of $_{29}Cu^{63}$ $Q = -1 \cdot 10^{-25}\ cm^2$ it is necessary according to formula (23) to ascribe to K the value 6, which, because of the small spin value of this nucleus, is not very probable, although large rotation values do figure in Guggenheimer's plan [10].

Another circumstance impelling us to be skeptical of the rotational effect is the following: because of the presence of non-central forces, the orbital moment of a number of motions in

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a complicated nucleus, just as its own spin, is not an integral motion.

The total moment of a number of motions is the total spin. It does not seem, therefore, very probable that there could be a rotational effect connected with the spin of the nucleus in those cases (nucleus with negative quadrupole moments) where a stronger effect is due to non-central forces and is also connected with the spin.

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